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## HELLENIC OPEN UNIVERSITY

## WROCLAW UNIVERSITY OF ECONOMICS

MASTER'S DEGREE PROGRAMME IN BUSINESS ADMINISTRATION

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Module: Financial Management and Accounting (MBA51)

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Second Written Assignment (WA2): Solutions


## Subject 1

## Alternative A

Initially we need to calculate the income statements for the 5-year span of the investment.

We assume that, besides depreciation expense and profit on sale of the ship, all other income also constitutes cash flows, realizable at year's end.

As such the income statements for years 1 to 5 are as follows:

| YEAR END |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INCOME STATEMENT | 0 | 1 | 2 | 3 | 4 | 5 |
| SALES |  | 10.800.000€ | 10.800.000 € | 10.800.000€ | 10.800.000 € | 10.800.000€ |
| COSTS |  | (6.500.000 €) | (6.500.000 €) | (6.500.000 €) | (6.500.000 €) | (6.500.000 €) |
| PROFIT / LOSS ON SALE OF ASSETS |  |  |  |  |  | 1.000 .000 € |
| DEPRECIATION |  | (2.600.000 €) | (2.600.000 €) | (2.600.000 €) | (2.600.000 €) | (2.600.000 €) |
| EBIT |  | $1.700 .000 €$ | $1.700 .000 €$ | 1.700.000 € | 1.700.000 € | 2.700 .000 € |
| INCOME TAX @ 35\% |  | (595.000 €) | (595.000 €) | (595.000 €) | (595.000 €) | (945.000 €) |
| NET OPERATING INCOME AFTER TAX |  | 1.105.000 € | 1.105.000 € | 1.105.000 € | 1.105.000 € | 1.755 .000 € |

Depreciation expense is calculated as follows:

$$
\begin{gathered}
\text { Depreciation }_{\text {annual }}=\frac{\text { Old Ship Total Initial Cost }- \text { Residual Value }}{\text { Economic life }} \\
=\frac{13.000 .000 €-0 €}{5 \text { years }}=2.600 .000 €
\end{gathered}
$$

So the profit on sale of the ship will be:
Profit on sale of ship year5 $=$ Expected selling price ${ }_{\text {year5 }}-$ Book value $_{\text {year5 }}$

$$
=1.000 .000 €-0 €=1.000 .000 €
$$

Income taxes amount to $35 \%$.

After concluding the income statements, and assuming that all items (exept depretiation and profit on sale of the ship) are also cash items, we can calculate the cash flows of each year by adding back to the net income the depretiation expense. Also, the initial cash flow (investment) at year 0 is the total cost of the ship (13.000.000€) and the investment in net working capital (1.000.000€). The latter, along with the further injections of $100.000 €$ each at the beginning of year 1 , year 2,

year 3 and year 4 will be released at the end of the project (end of year 5), thus resulting in an inflow of 1.400.000€.

We expect that the ship is sold as scrap at the end of year 5 for $1.000 .000 €$.
As such, the project's cash flows are formed as follows:

| YEAR END |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RELEVANT CASH FLOWS | 0 | 1 | 2 | 3 | 4 | 5 |  |
| INTIAL INVESTMENT IN OLD SHIP | $(13.000 .000 €)$ |  |  |  |  |  |  |
| SALE OF PURCHASED SHIP |  |  |  |  |  |  |  |
| NET WORKING CAPITAL INVESTMENT / DISINVESTMENT | $(1.000 .000 €)$ | $(100.000 €)$ | $(100.000 €)$ | $(100.000 €)$ | $(100.000 €)$ | $1.400 .000 €$ |  |
| PROJECT CASH FLOWS |  | $3.705 .000 €$ | $3.705 .000 €$ | $3.705 .000 €$ | $3.705 .000 €$ | $3.355 .000 €$ |  |
| TOTAL CASH FLOWS | $(14.000 .000 €)$ | $3.605 .000 €$ | $3.605 .000 €$ | $3.605 .000 €$ | $3.605 .000 €$ | $5.755 .000 €$ |  |

## Calculation of the WACC:

The initial investment commands $14.000 .000 €(13.000 .000 €$ for the purchase of the old ship plus $1.000 .000 €$ in working capital). The issuance of a nominal value of 14.000.000€ of preferred stock does not suffice due to the floating costs $(1,50 \%$ of nominal value). As such, another $210.000 €$ (net of flotation costs) will be needed and this will come from the issuance of a bond with face value of $213.198 €$ (minus flotation costs of 1,50\% will lead to a net amount of 210.000€).

The cost of the bond is as follows:

$$
\text { After }- \text { tax } \operatorname{cost}_{\text {bond }}=\mathrm{k}_{\text {bond }} \times(1-\text { tax rate })=10,40 \% \times(1-35 \%)=6,76 \%
$$

Where the pre-tax cost of bond ( $\mathrm{k}_{\text {bond }}$ ) is the real yield to maturity (i.e. if we consider as present value the net proceeds of the bond) and is calculated with the RATE function in MS-Excel:

Net proceeds $\mathrm{b}_{\text {bond }}=\sum_{\mathrm{t}=1}^{5} \frac{\text { coupon }}{\left(1+\mathrm{k}_{\text {bond }}\right)^{\mathrm{t}}}+\frac{\text { face value }}{\left(1+\mathrm{k}_{\mathrm{bond}}\right)^{5}} \stackrel{\text { trial and error }}{\Longrightarrow} \mathrm{k}_{\text {bond }}=10,40 \%$

The cost of the preferred shares is as follows:

$$
\mathrm{k}_{\mathrm{PS}}=\frac{\text { Preferred dividend per share }}{\text { Net issue price today }}=\frac{0,50 €}{5 € \times(1-1,50 \%)}=10,15 \%
$$



So:

$$
\begin{aligned}
W A C C=\text { After } & - \text { tax cost } \\
\text { bond } & \times \text { weight }_{\text {bond }}+\mathrm{k}_{\text {PS }} \times \text { weight }_{\text {preferred stock }} \\
& =10,15 \% \times \frac{13.790 .000}{14.000 .000}+6,76 \% \times \frac{210.000}{14.000 .000}=10,10 \%
\end{aligned}
$$

The present value of the cash flow for years 1 to 5 , at $10,10 \%$ discount rate, is:

$$
\mathrm{PV}=\sum_{\mathrm{t}=1}^{5} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{r})^{\mathrm{t}}}=14.959 .305 €
$$

$$
\begin{aligned}
& \mathrm{NPV}=\mathrm{PV}-\text { Initial investment }=\mathrm{PV}-\mathrm{CF}_{0}=14.959 .305 €-14.000 .000 € \\
& =959.305 €
\end{aligned}
$$

IRR is the discount rate which when applied leads to zero NPV. That is:

$$
\mathrm{NPV}=0 \Rightarrow \mathrm{PV}-\mathrm{CF}_{0}=0 \Rightarrow \sum_{\mathrm{t}=1}^{5} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{IRR})^{\mathrm{t}}}-\mathrm{CF}_{0}=0 \Rightarrow \mathrm{IRR}=12,58 \%
$$

The profitability index $(\mathrm{PI})$ is calculated as follows:

$$
\mathrm{PI}=\frac{\mathrm{PV}}{\mathrm{CF}_{0}}=\frac{14.959 .305 €}{14 \cdot 000.000 €}=1,07
$$

The project is not to be rejected as a standalone project, since:
NPV=959.305 $€>0$
IRR=12,58\%>10,10\%=WACC
$\mathrm{PI}=1,07>1$

## Alternative B

(the initial investment in working capital of 1 mil euros takes place @ year1)


Initially we need to calculate the income statements for the 5-year span of the investment.

We assume that, besides depreciation expense and profit on sale of the ship, all other income also constitutes cash flows, realizable at year's end.

As such the income statements for years 1 to 5 are as follows:

| YEAR END |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INCOME STATEMENT | 0 | 1 | 2 | 3 | 4 | 5 |
| SALES |  | $12.000 .000 €$ | $12.000 .000 €$ | $12.000 .000 €$ | $12.000 .000 €$ | $12.000 .000 €$ |
| COSTS |  | (6.400.000 €) | (6.400.000 €) | (6.400.000 €) | (6.400.000 €) | (6.400.000 €) |
| PROFIT / LOSS ON SALE OF ASSETS |  |  |  |  |  | $1.000 .000 €$ |
| DEPRECIATION |  | (3.300.000 €) | (3.300.000 €) | (3.300.000 €) | (3.300.000 €) | (3.300.000 €) |
| EBIT |  | $2.300 .000 €$ | 2.300.000 € | 2.300.000 € | 2.300.000 € | 3.300.000 € |
| INCOME TAX @ 35\% |  | (805.000 €) | (805.000 €) | (805.000 €) | (805.000 €) | $(1.155 .000$ €) |
| NET OPERATING INCOME AFTER TAX |  | 1.495.000 € | 1.495.000 € | 1.495.000€ | 1.495.000 € | 2.145.000€ |

Depreciation expense is calculated as follows:
Depreciation $_{\text {annual }}=\frac{\text { Old Ship Total Initial Cost }+ \text { Upgrade cost }- \text { Residual Value }}{\text { Economic life }}$

$$
=\frac{13.000 .000 €+3.500 .000 €-0 €}{5 \text { years }}=3.300 .000 €
$$

So the profit on sale of the ship will be:
Profit on sale of ship year5 $=$ Expected selling price ${ }_{\text {year } 5}-$ Book value $_{\text {year5 }}$

$$
=1.000 .000 €-0 €=1.000 .000 €
$$

Income taxes amount to 35\%.

After concluding the income statements, and assuming that all items (exept depretiation and profit on sale of the ship) are also cash items, we can calculate the cash flows of each year by adding back to the net income the depretiation expense. Also, the initial cash flow (investment) at year 0 is the total cost of the ship (13.000.000€) and the upgrade cost $(3.500 .000 €)$. The investment in net working capital $(1.000 .000 €)$ takes place at the end of year 1 . The latter, along with the further injections of $120.000 €$ each at the end of year 2 , year 3 and year 4 will be released at the end of the project (end of year 5), thus resulting in an inflow of 1.360.000€.


We expect that the ship is sold as scrap at the end of year 5 for 1.000.000€.
As such, the project's cash flows are formed as follows:

| YEAR END |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RELEVANT CASH FLOWS | 0 | 1 | 2 | 3 | 4 | 5 |
| INITIAL INVESTMENT IN OLD SHIP | $(13.000 .000 €)$ |  |  |  |  |  |
| UPGRADE COST | (3.500.000 €) |  |  |  |  |  |
| SALE OF PURCHASED SHIP |  |  |  |  |  | 1.000.000 € |
| NET WORKING CAPITAL INVESTMENT / DISINVESTMENT | $0 €$ | (1.000.000 €) | (120.000€) | (120.000€) | (120.000€) | 1.360 .000 € |
| PROJECT CASH FLOWS |  | 4.795.000€ | $4.795 .000 €$ | $4.795 .000 €$ | 4.795 .000 € | 4.445 .000 € |
| TOTAL CASH FLOWS | (16.500.000 €) | $3.795 .000 €$ | 4.675.000€ | $4.675 .000 €$ | 4.675.000€ | 6.805.000 € |

## Calculation of the WACC:

The initial investment commands $16.500 .000 €(13.000 .000 €$ for the purchase of the old ship plus $3.500 .000 €$ the upgrade). The issuance of a nominal value of $14.000 .000 €$ of preferred stock does not suffice. Due to the floating costs $(1,50 \%$ of nominal value) the proceeds from the issuance of the preferred stock is $13.790 .000 €$. As such, another 2.710.000€ (net of flotation costs) will be needed and this will come from the issuance of a bond with face value of 2.751.269€ (minus flotation costs of $1,50 \%$ will lead to a net amount of $2 \cdot 710.000 €$ ).

The cost of the bond is as follows:

$$
\text { After }-\operatorname{tax} \operatorname{cost}_{\text {bond }}=\mathrm{k}_{\text {bond }} \times(1-\operatorname{tax} \text { rate })=10,40 \% \times(1-35 \%)=6,76 \%
$$

Where the pre-tax cost of bond ( $\mathrm{k}_{\text {bond }}$ ) is the real yield to maturity (i.e. if we consider as present value the net proceeds of the bond) and is calculated with the RATE function in MS-Excel:

Net proceeds ${ }_{\text {bond }}=\sum_{\mathrm{t}=1}^{5} \frac{\text { coupon }}{\left(1+\mathrm{k}_{\text {bond }}\right)^{\mathrm{t}}}+\frac{\text { face value }}{\left(1+\mathrm{k}_{\text {bond }}\right)^{5}} \stackrel{\text { trial and error }}{ } \mathrm{k}_{\text {bond }}=10,40 \%$

The cost of the preferred shares is as follows:

$$
\mathrm{k}_{\mathrm{PS}}=\frac{\text { Preferred dividend per share }}{\text { Net issue price today }}=\frac{0,50 €}{5 € \times(1-1,50 \%)}=10,15 \%
$$



So:

$$
\begin{aligned}
W A C C=\text { After } & - \text { tax } \text { cost }_{\text {bond }} \times \text { weight }_{\text {bond }}+\mathrm{k}_{\text {PS }} \times \text { weight }_{\text {preferred stock }} \\
& =10,15 \% \times \frac{13.790 .000}{16.500 .000}+6,76 \% \times \frac{3.710 .000}{16.500 .000}=9,60 \%
\end{aligned}
$$

The present value of the cash flow for years 1 to 5 , at $9,60 \%$ discount rate, is:

$$
\mathrm{PV}=\sum_{\mathrm{t}=1}^{5} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{r})^{\mathrm{t}}}=18.450 .993 €
$$

$$
\mathrm{NPV}=\mathrm{PV}-\text { Initial investment }=\mathrm{PV}-\mathrm{CF}_{0}=18.450 .993 €-16.500 .000 €
$$

$$
=1.950 .993 €
$$

IRR is the discount rate which when applied leads to zero NPV. That is:

$$
\mathrm{NPV}=0 \Rightarrow \mathrm{PV}-\mathrm{CF}_{0}=0 \Rightarrow \sum_{\mathrm{t}=1}^{5} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{IRR})^{\mathrm{t}}}-\mathrm{CF}_{0}=0 \Rightarrow \mathrm{IRR}=13,73 \%
$$

The profitability index (PI) is calculated as follows:

$$
\mathrm{PI}=\frac{\mathrm{PV}}{\mathrm{CF}_{0}}=\frac{18.450 .993 €}{16 \cdot 500 \cdot 000 €}=1,12
$$

The project is not to be rejected as a standalone project, since:
NPV=1.950.993€>0
IRR=13,73\%>9,60\%=WACC
$\mathrm{Pl}=1,12>1$

## Alternative C

Initially we need to calculate the income statements for the 5 -year span of the investment.

We assume that, besides depreciation expense and profit on sale of the ship, all other income also constitutes cash flows, realizable at year's end.


As such the income statements for years 1 to 5 are as follows:

| YEAR END |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INCOME STATEMENT | 0 | 1 | 2 | 3 | 4 | 5 |
| SALES |  | 15.000.000€ | 15.000.000€ | 15.000.000€ | 15.000.000€ | 15.000.000€ |
| COSTS |  | (8.900.000 €) | (8.900.000 €) | (8.900.000 €) | (8.900.000 €) | (8.900.000€) |
| PROFIT / LOSS ON SALE OF ASSETS |  |  |  |  |  | 8.000.000€ |
| DEPRECIATION |  | (4.400.000 €) | (4.400.000 €) | (4.400.000 €) | (4.400.000 €) | (4.400.000 €) |
| EBIT |  | 1.700.000€ | 1.700.000€ | 1.700.000€ | 1.700.000€ | 9.700.000€ |
| INCOME TAX @ 35\% |  | (595.000€) | (595.000€) | (595.000€) | (595.000 €) | (3.395.000€) |
| NET OPERATING INCOME AFTER TAX |  | 1.105.000 € | 1.105.000€ | 1.105.000€ | 1.105.000€ | 6.305 .000 € |

Depreciation expense is calculated as follows:

$$
\begin{gathered}
\text { Depreciation }_{\text {annual }}=\frac{\text { New Ship Total Initial Cost }- \text { Residual Value }}{\text { Economic life }} \\
=\frac{22.000 .000 €-0 €}{5 \text { years }}=4.400 .000 €
\end{gathered}
$$

So the profit on sale of the ship will be:
Profit on sale of ship year5 $=$ Expected selling price $_{\text {year5 }}-$ Book value $_{\text {year5 }}$

$$
=8.000 .000 €-0 €=8.000 .000 €
$$

Income taxes amount to $35 \%$.

After concluding the income statements, and assuming that all items (exept depretiation and profit on sale of the ship) are also cash items, we can calculate the cash flows of each year by adding back to the net income the depretiation expense. Also, the initial cash flow (investment) at year 0 is the total cost of the ship (22.000.000€) and the investment in net working capital (2.000.000€). The latter, along with the further injections of $200.000 €$ each at the beginning of year 1 , year 2, year 3 and year 4 will be released at the end of the project (end of year 5), thus resulting in an inflow of 2.800.000€.

We expect that the ship is sold at the end of year 5 for $8.000 .000 €$.
As such, the project's cash flows are formed as follows:

| YEAR END |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RELEVANT CASH FLOWS | 0 | 1 | 2 | 3 | 4 | 5 |
| INTITAL INVESTMENT IN NEW SHP | (22.000.000€) |  |  |  |  |  |
| SALE OF PURCHASED SHP |  |  |  |  |  | 8.000.000€ |
| NET WORKING CAPTTAL INVESTMENT / DISINVESTMENT | (2.000.000€) | (200.000 €) | (200.000€) | (200.000€) | (200.000€) | 2.800.000€ |
| PROJECT CASH FLOWS |  | 5.505.000€ | 5.505.000€ | 5.505.000€ | 5.505.000€ | 2.705.000€ |
| TOTAL CASH FLOWS | (24,000,000 €) | 5.305.000€ | 5.305.000€ | 5.305.000€ | 5.305.000€ | 13.505.000€ |

## Calculation of the WACC:

The initial investment commands $24.000 .000 €(22.000 .000 €$ for the purchase of the new ship plus $2.000 .000 €$ in working capital). The issuance of a nominal value of $14.000 .000 €$ of preferred stock does not suffice. Due to the floating costs $(1,50 \%$ of nominal value) the proceeds from the issuance of the preferred stock is 13.790.000€. As such, another $10.210 .000 €$ (net of flotation costs) will be needed and this will come from the issuance of a bond with face value of 10.365.482€ (minus flotation costs of $1,50 \%$ will lead to a net amount of 10.210.000€).

The cost of the bond is as follows:
After - tax $\operatorname{cost}_{\text {bond }}=\mathrm{k}_{\text {bond }} \times(1-$ tax rate $)=10,40 \% \times(1-35 \%)=6,76 \%$
Where the pre-tax cost of bond ( $\mathrm{k}_{\text {bond }}$ ) is the real yield to maturity (i.e. if we consider as present value the net proceeds of the bond) and is calculated with the RATE function in MS-Excel:

Net proceeds ${ }_{\text {bond }}=\sum_{\mathrm{t}=1}^{5} \frac{\text { coupon }}{\left(1+\mathrm{k}_{\text {bond }}\right)^{\mathrm{t}}}+\frac{\text { face value }}{\left(1+\mathrm{k}_{\text {bond }}\right)^{5}} \stackrel{\text { trial and error }}{ } \mathrm{k}_{\text {bond }}=10,40 \%$

The cost of the preferred shares is as follows:

$$
\mathrm{k}_{\mathrm{PS}}=\frac{\text { Preferred dividend per share }}{\text { Net issue price today }}=\frac{0,50 €}{5 € \times(1-1,50 \%)}=10,15 \%
$$



So:

$$
\begin{aligned}
& W A C C=\text { After }- \text { tax }_{\text {cost }}^{\text {bond }} \\
& \times \text { weight }_{\text {bond }}+\mathrm{k}_{\text {PS }} \times \text { weight }_{\text {preferred stock }} \\
&=10,15 \% \times \frac{13.790 .000}{24.000 .000}+6,76 \% \times \frac{10.210 .000}{24.000 .000}=8,71 \%
\end{aligned}
$$

The present value of the cash flow for years 1 to 5 , at $8,71 \%$ discount rate, is:

$$
\mathrm{PV}=\sum_{\mathrm{t}=1}^{5} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{r})^{\mathrm{t}}}=26.192 .437 €
$$

$$
\mathrm{NPV}=\mathrm{PV}-\text { Initial investment }=\mathrm{PV}-\mathrm{CF}_{0}=26.192 .437 €-24.000 .000 €
$$

$$
=2.192 .437 €
$$

IRR is the discount rate which when applied leads to zero NPV. That is:

$$
\mathrm{NPV}=0 \Rightarrow \mathrm{PV}-\mathrm{CF}_{0}=0 \Rightarrow \sum_{\mathrm{t}=1}^{5} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{IRR})^{\mathrm{t}}}-\mathrm{CF}_{0}=0 \Rightarrow \mathrm{IRR}=11,67 \%
$$

The profitability index (PI) is calculated as follows:

$$
\mathrm{PI}=\frac{\mathrm{PV}}{\mathrm{CF}_{0}}=\frac{26 \cdot 192.437 €}{24.000 .000 €}=1,09
$$

The project is not to be rejected as a standalone project, since:
$N P V=2.192 .437 €>0$
IRR=11,67\%>8,71\%=WACC
$\mathrm{PI}=1,09>1$

The optimal alternative is C, since it leads to a higher NPV value. When appraising mutual exclusive projects NPV point to the optimal project. That is because NPV represents the net wealth (in currency units) that each project contributes to the firm.


## Subject 2

Initially, we need to calculate the INCREMENTAL ${ }^{1}$ income statements for the 2-year span of the new project (creating a new bottle of $0,25 \mathrm{lt}$ ).

We assume that, besides depreciation expense and profit or loss on the sale of the new machinery employed in the project (if any), all other income also constitutes cash flows, realizable at year's end.

As such, for Scenario A, the income statements for years 1 to 2 are as follows:

|  | YEAR END |  |  |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
|  |  | $\mathbf{0}$ | $\mathbf{1}$ |
| INCOME STATEMENT |  | $225.000 €$ | $360.000 €$ |
| SALES |  | $(120.000 €)$ | $(200.000 €)$ |
| VARIABLE COSTS |  | $(20.000 €)$ | $(25.000 €)$ |
| FIXED COSTS |  | $30.000 €$ |  |
| PROFIT / LOSS ON SALE OF ASSETS |  | $(41.000 €)$ | $(41.000 €)$ |
| DEPRECIATION |  | $\mathbf{4 4 . 0 0 0} €$ | $\mathbf{1 2 4 . 0 0 0} €$ |
| EBIT |  | $(11.000 €)$ | $(31.000 €)$ |
| INCOME TAX @ 25\% | $\mathbf{3 3 . 0 0 0} €$ | $\mathbf{9 3 . 0 0 0} €$ |  |
| NET OPERATING INCOME AFTER TAX |  |  |  |

The Sales for the new product in each year are calculated

- 150.000 items $\times 1,5 €$ per unit $=225.000 €$ for the $1^{\text {st }}$ year
- 200.000 items $\times 1,8 €$ per unit $=360.000 €$ for the $2^{\text {nd }}$ year

The variable costs for the new product in each year are calculated

- 150.000 items $\times(-0,8 €)$ per unit $=-120.000 €$ for the $1^{\text {st }}$ year
- 200.000 items $x(-1 €)$ per unit $=-200.000 €$ for the $2^{\text {nd }}$ year

The fixed costs (i.e. administrative and marketing expenses) for the new product in each year are as follows

- $-20.000 €$ for the $1^{\text {st }}$ year
- $-25.000 €$ for the $2^{\text {nd }}$ year

[^0]

The cost of the market research performed is ignored, since, this is a sunk cost, already undertaken, thus, it is irrelevant whether the firm will go on with the project or not. As such, it not an incremental cost.

For tax purposes, the $82.000 €$ investment in machinery (which includes the cost of trasportation and installation) will be fully depreciated, lineraly during the 2-years economic life. As such, the annual depreciation is:

$$
\text { Depreciation }=\frac{\text { Asset cost }- \text { Residual value }}{\text { Useful life }}=\frac{82.000 €-0 €}{2 \text { years }}=41.000 €
$$

For the Scenario A, the cash flows of the project are as follows:

|  | YEAR END |  |  |
| :--- | :---: | :---: | ---: |
| RELEVANT CASH FLOWS | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| INITIAL INVESTMENT IN NEW MACHINERY | $(82.000 €)$ |  |  |
| SALE OF NEW MACHINERY |  |  | $30.000 €$ |
| NET WORKING CAPITAL INVESTMENT / DISINVESTMENT | $(15.000 €)$ | $(2.000 €)$ | $17.000 €$ |
| PROJECT CASH FLOWS |  | $74.000 €$ | $104.000 €$ |
| TOTAL CASH FLOWS | $(97.000 €)$ | $\mathbf{7 2 . 0 0 0} €$ | $\mathbf{1 5 1 . 0 0 0} €$ |

The machinery can be sold at the end of year 2 for $30.000 €$.
We assume that any investment in working capital takes place at the beginning of the year. As such, since the working capital of year 1 is $15.000 €$, the firm incures a cash outflow of $15.000 €$ today (beginning of year 1). And since the working capital of year 2 is $17.000 €$, the firm incures an extra $2.000 €$ cash outflow at the end of year 1 (beginning of year 2). Finally, since the working capital associated with the project will be recovered at the end of year 2 , the firm will benefit a cash inflow of $17.000 €$ at the end of year 2 .

The project cash flows for any given year are calculated by adding back the depreciation expenses to the net operating profit after tax, as follows:

Net Operating Profit after tax + Depreciation

Now, for Scenario B, the income statements for years 1 to 2 are as follows:


| YEAR END |  |  |  |
| :---: | :---: | :---: | :---: |
| INCOME STATEMENT | 0 | 1 | 2 |
| SALES |  | 340.000 € | 500.000 € |
| VARIABLE COSTS |  | (200.000 €) | (300.000 €) |
| FIXED COSTS |  | (25.000 €) | (30.000 €) |
| PROFIT / LOSS ON SALE OF ASSETS |  |  | 30.000 € |
| DEPRECIATION |  | $(41.000$ €) | $(41.000$ €) |
| EBIT |  | 74.000 € | 159.000 € |
| INCOME TAX @ 25\% |  | (18.500 €) | (39.750 €) |
| NET OPERATING INCOME AFTER TAX |  | 55.500 € | 119.250 € |

The Sales for the new product in each year are calculated

- 200.000 items $\times 1,7 €$ per unit $=340.000 €$ for the $1^{\text {st }}$ year
- 250.000 items $\times 2 €$ per unit $=500.000 €$ for the $2^{\text {nd }}$ year

The variable costs for the new product in each year are calculated

- 200.000 items $\times(-1 €)$ per unit $=-200.000 €$ for the $1^{\text {st }}$ year
- 250.000 items $\times(-1,2 €)$ per unit $=-300.000 €$ for the $2^{\text {nd }}$ year

The fixed costs (i.e. administrative and marketing expenses) for the new product in each year are as follows

- $-25.000 €$ for the $1^{\text {st }}$ year
- -30.000€ for the $2^{\text {nd }}$ year

The cost of the market research performed is ignored, since, this is a sunk cost, already undertaken, thus, it is irrelevant whether the firm will go on with the project or not. As such, it not an incremental cost.

For tax purposes, the $82.000 €$ investment in machinery (which includes the cost of trasportation and installation) will be fully depreciated, lineraly during the 2-years economic life. As such, the annual depreciation is:

$$
\text { Depreciation }=\frac{\text { Asset cost }- \text { Residual value }}{\text { Useful life }}=\frac{82.000 €-0 €}{2 \text { years }}=41.000 €
$$

For the Scenario B, the cash flows of the project are as follows:

|  | YEAR END |  |  |
| :--- | :---: | :---: | ---: |
|  |  |  |  |
| RELEVANT CASH FLOWS | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| INITIAL INVESTMENT IN NEW MACHINERY | $(82.000 €)$ |  |  |
| SALE OF NEW MACHINERY |  |  | $30.000 €$ |
| NET WORKING CAPITAL INVESTMENT / DISINVESTMENT | $(15.000 €)$ | $(2.000 €)$ | $17.000 €$ |
| PROJECT CASH FLOWS |  | $96.500 €$ | $130.250 €$ |
| TOTAL CASH FLOWS | $(97.000 €)$ | $\mathbf{9 4 . 5 0 0 €}$ | $\mathbf{1 7 7 . 2 5 0} €$ |

The machinery can be sold at the end of year 2 for $30.000 €$.
We assume that any investment in working capital takes place at the beginning of the year. As such, since the working capital of year 1 is $15.000 €$, the firm incures a cash outflow of $15.000 €$ today (beginning of year 1). And since the working capital of year 2 is $17.000 €$, the firm incures an extra $2.000 €$ cash outflow at the end of year 1 (beginning of year 2). Finally, since the working capital associated with the project will be recovered at the end of year 2 , the firm will benefit a cash inflow of $17.000 €$ at the end of year 2 .

The project cash flows for any given year are calculated by adding back the depreciation expenses to the net operating profit after tax, as follows:

Net Operating Profit after tax + Depreciation

Taking into consideration that Scenario A is assigned with $45 \%$ probability and Scenario B with $55 \%$ probability, the expected cash flows of the project for each year are calculated as follows:

$$
\mathrm{E}\left(\mathrm{CF}_{\mathrm{t}}\right)=\sum_{\mathrm{i}} \mathrm{p}_{\mathrm{i}} \times \mathrm{CF}_{\mathrm{t}, \mathrm{i}}
$$

Where:
$\mathrm{E}\left(\mathrm{CF}_{\mathrm{t}}\right) \quad$ the expected cash flow of year $\mathrm{t}, \mathrm{t}=0,1,2$
$p_{i} \quad$ the probab;ility of Scenario $i, i=A, B$
$\mathrm{CF}_{\mathrm{t}, \mathrm{i}} \quad$ the cash flow of year t under the Scenario i
As such:

$$
\begin{gathered}
\mathrm{E}\left(\mathrm{CF}_{0}\right)=45 \% \times(-97.000 €)+55 \% \times(-97.000 €)=-97.000 € \\
\mathrm{E}\left(\mathrm{CF}_{1}\right)=45 \% \times 72.000 €+55 \% \times 94.500 €=84.375 € \\
\mathrm{E}\left(\mathrm{CF}_{2}\right)=45 \% \times 151.000 €+55 \% \times 177.250 €=165.438 €
\end{gathered}
$$



Accordingly, the standard deviation for each year is calculated as follows:

$$
\sigma_{\mathrm{CF}_{\mathrm{t}}}=\sqrt{\sum_{\mathrm{i}} \mathrm{p}_{\mathrm{i}} \times\left[\mathrm{CF}_{\mathrm{t}, \mathrm{i}}-\mathrm{E}\left(\mathrm{CF}_{\mathrm{t}}\right)\right]^{2}}
$$

As such:

$$
\begin{aligned}
& \sigma_{\mathrm{CF}_{0}} \\
& =\sqrt{45 \% \times\left[(-97.000 €-(-97.000 €)]^{2}+55 \% \times\left[(-97.000 €-(-97.000 €)]^{2}\right.\right.} \\
& =0 € \\
& \sigma_{\mathrm{CF}_{1}}=\sqrt{45 \% \times[72.000 €-84.375 €]^{2}+55 \% \times[94.500 €-84.375 €]^{2}} \\
& =11.194 € \\
& \sigma_{\mathrm{CF}_{2}}=\sqrt{45 \% \times[151.000 €-165.438 €]^{2}+55 \% \times[177.250 €-165.438 €]^{2}} \\
& =13.059 €
\end{aligned}
$$

Thus, the coefficient of variation for each year is calculated as follows:

$$
\begin{gathered}
\mathrm{CV}_{0}=\frac{\sigma_{\mathrm{CF}_{0}}}{\mathrm{E}\left(\mathrm{CF}_{0}\right)}=\frac{0 €}{-97.000 €}=0 \% \\
\mathrm{CV}_{1}=\frac{\sigma_{\mathrm{CF}_{1}}}{\mathrm{E}\left(\mathrm{CF}_{1}\right)}=\frac{11.194 €}{84.375 €}=13,27 \% \\
\mathrm{CV}_{2}=\frac{\sigma_{\mathrm{CF}_{2}}}{\mathrm{E}\left(\mathrm{CF}_{2}\right)}=\frac{13.059 €}{165.438 €}=7,89 \%
\end{gathered}
$$

Year 2, although it exhibits a higher volatility (i.e. standard deviation) in cash flows in nominal terms, however, in relative terms, it is less risky, as the coefficient of variation exposes.

All the above are summarized in the table below:

| YEAR END |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| Scenario A's Total Cash Flows | $(97.000 €)$ | $72.000 €$ | $151.000 €$ |
| Probability of Scenario A | $45,00 \%$ | $45,00 \%$ | $45,00 \%$ |
| Scenario B's Total Cash Flows | $(97.000 €)$ | $94.500 €$ | $177.250 €$ |
| Probability of Scenario B | $55,00 \%$ | $55,00 \%$ | $55,00 \%$ |
| EXPECTED TOTAL CASH FLOWS | $(97.000 €)$ | $84.375 €$ | $\mathbf{1 6 5 . 4 3 8} €$ |
| STANDARD DEVIATION | $\mathbf{0} €$ | $\mathbf{1 1 . 1 9 4 €}$ | $\mathbf{1 3 . 0 5 9} €$ |
| C.V. | $\mathbf{0 , 0 0 \%}$ | $\mathbf{1 3 , 2 7 \%}$ | $\mathbf{7 , 8 9 \%}$ |

Now we can calculate the present values of expected cash flows and evaluate the net present value.


The present value of the cash flow for years 1 to 2, at 10\% discount rate is:

$$
\mathrm{PV}=\sum_{\mathrm{t}=1}^{2} \frac{\mathrm{E}\left(\mathrm{CF}_{\mathrm{t}}\right)}{(1+\mathrm{WACC})^{\mathrm{t}}}=\frac{84.375 €}{(1+10 \%)^{1}}+\frac{165.438 €}{(1+10 \%)^{2}}=213.430 €
$$

$$
\mathrm{NPV}=\mathrm{PV}-\text { Initial investment }=\mathrm{PV}-\mathrm{CF}_{0}=213.430 €-97.000 €=116.430 €
$$

Since NPV>0, the project is accepted.

Net present value and IRR are based on the same equation:

$$
\mathrm{NPV}=\sum_{\mathrm{t}=1}^{\mathrm{T}} \frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{r})^{\mathrm{t}}}-\text { Initial investment }
$$

IRR is the rate $r$ that will lead to $N P V=0$
As such, the Net present value and IRR will always lead to the same decisions when applied to independent investment projects.

However, when it comes to mutually exclusive projects, they will not always rank the appraised projects in the same order. This is because IRR is an interest rate. On the other hand, NPV shows us the real wealth (in monetary terms, not percentage) that any project will contribute to the firm.

## Example:

Let us assume that we have to decide on the mutually exclusive projects $Y$ and $Z$ as follows:

|  | WACC | 9,00\% |
| :---: | :---: | :---: |
|  | Cash Flows |  |
| Years | Project $\mathbf{Y}$ | Project Z |
| 0 | $(120.000,00 €)$ | $(120.000,00 €)$ |
| 1 | $80.000,00 €$ | $5.000,00 €$ |
| 2 | $50.000,00 €$ | $5.000,00 €$ |
| 3 | $10.000,00 €$ | $10.000,00 €$ |
| 4 |  | $20.000,00 €$ |
| 5 |  | $25.000,00 €$ |
| 6 |  | $25.000,00 €$ |
| 7 |  | $30.000,00 €$ |
| 8 |  | $30.000,00 €$ |
| 9 |  | $35.000,00 €$ |
| 10 |  | $35.000,00 €$ |


|  | Project $\mathbf{Y}$ | Project Z |
| :--- | :---: | :---: |
| Payback period | 1,80 | 7,00 |
| Discounted payback period | 2,59 | 9,72 |
| NPV | $3.200,33 €$ | $4.207,23 €$ |
| PI | 1,03 | 1,04 |
| IRR | $10,98 \%$ | $9,59 \%$ |

When confronting with mutually exclusive projects it is safer to go with the NPV criterion. The NPV rule is calculating the actual wealth that a project adds to our company in real currency terms, not as a percentage.

In our case, while project $Y$ leads to an IRR of $10,98 \%$, its NPV (thus the added wealth) is $3.200,33 €$. On the other hand, project $Z$ though providing a thinner return ( $9,59 \%$ as compared to $10,98 \%$ of project $Y$ ), it does contribute $4.207,23 €$ to the value of the company, around $900 €$ more than project Y .

As such, it is advisable to go with project $Z$.


## Subject 3

## Part A

The cost of the zero-coupon bond is:

|  | coupon rate | 0,00\% |
| :---: | :---: | :---: |
|  | coupons per annum | 1 |
|  | maturity | 5 |
|  | face value | 32.000.000,00 € |
|  | market value | 25.600.000,00 € |
|  | yield to maturity (cost of debt) | 4,56\% |
|  | after tax cost of debt | 2,97\% |

$$
\begin{aligned}
& \text { After }- \text { tax } \text { cost }_{\text {zero-coupon bond }}=\mathrm{k}_{\text {zero-coupon bond }} \times(1-\text { tax rate }) \\
& \quad=4,56 \% \times(1-35 \%)=2,97 \%
\end{aligned}
$$

Where:

$$
\mathrm{k}_{\text {zero }- \text { coupon bond }}=\text { cost of debt }=\text { yield to maturity }
$$

And it is calculated with trial and error from the following equation:

$$
\begin{gathered}
P_{0}=\sum_{t=1}^{T \times m} \frac{C}{\left(1+\frac{k_{\text {zero-coupon bond }}^{m}}{m}\right)^{t}}+\frac{F V}{\left(1+\frac{\left.k_{\text {zero-coupon bond }}\right)^{T \times m}}{m}\right.} \\
P_{\mathrm{t}=1}^{\mathrm{T} \times \mathrm{m}} \sum_{\left(1+\frac{\mathrm{k}_{\text {zero-coupon bond }}}{\mathrm{m}}\right)^{\mathrm{t}}}+\frac{\mathrm{C}}{\left(1+\frac{\left.\mathrm{k}_{\text {zero-coupon bond }}\right)^{\mathrm{T} \times \mathrm{m}}}{\mathrm{~m}}\right.} \\
=\sum_{\mathrm{t}=1}^{5 \times 1} \frac{0 €}{\left(1+\frac{\left.\mathrm{k}_{\text {zero-coupon bond }}\right)^{\mathrm{t}}}{1}+\frac{32.000 .000 €}{\left(1+\frac{\mathrm{k}_{\text {zero-coupon bond }}}{1}\right)^{5 \times 1}}\right.} \\
\Rightarrow 25.600 .000 €=\frac{32.000 .000 €}{\left(1+\mathrm{k}_{\text {zero-coupon bond }}\right)^{5}} \Rightarrow \\
\mathrm{k}_{\text {zero-coupon bond }}=\sqrt[5]{\frac{32.000 .000 €}{25.600 .000 €}}-1=4,56 \%
\end{gathered}
$$

Where

$P_{0}=25.600 .000 €$
$\mathrm{T}=5$
$F V=32.000 .000 €:$
$\mathrm{i}=0 \%$
the market value of the zero-coupon bond the bond's maturity in years
the face value of the bond
the coupon rate

The cost of the bond is as follows:

| coupon rate | $4,00 \%$ |
| :--- | ---: |
| coupons per annum | 1 |
| maturity | 5 |
| face value | $26.000 .000,00 €$ |
| market value | $24.700 .000,00 €$ |
| yield to maturity (cost of debt) | $\mathbf{5 , 1 6 \%}$ |
| after tax cost of debt | $\mathbf{3 , 3 5 \%}$ |

After - tax cost ${ }_{\text {bond }}=\mathrm{k}_{\text {bond }} \times(1-$ tax rate $)=5,16 \% \times(1-35 \%)=3,35 \%$
Where:

$$
\mathrm{k}_{\mathrm{bond}}=\text { cost } \text { of debt }=\text { yield to maturity }
$$

And it is calculated with trial and error from the following equation:

$$
\begin{gathered}
P_{0}=\sum_{t=1}^{T \times m} \frac{C}{\left(1+\frac{k_{\text {bond }}}{m}\right)^{t}}+\frac{F V}{\left(1+\frac{k_{\text {bond }}}{m}\right)^{T \times m}} \\
\mathrm{P}_{0}=\sum_{\mathrm{t}=1}^{\mathrm{T} \times \mathrm{m}} \frac{\mathrm{C}}{\left(1+\frac{\mathrm{k}_{\text {bond }}}{\mathrm{m}}\right)^{\mathrm{t}}}+\frac{\mathrm{FV}}{\left(1+\frac{\mathrm{k}_{\text {bond }}}{\mathrm{m}}\right)^{\mathrm{T} \times \mathrm{m}}}=\sum_{\mathrm{t}=1}^{5 \times 1} \frac{1.040 .000 €}{\left(1+\frac{\mathrm{k}_{\text {bond }}}{1}\right)^{\mathrm{t}}}+\frac{26.000 .000 €}{\left(1+\frac{\mathrm{k}_{\text {bond }}}{1}\right)^{5 \times 1}} \\
=24.700 .000 €
\end{gathered}
$$

$$
\xrightarrow{\text { through trial and error }} \mathrm{k}_{\text {bond }}=5,16 \%
$$

Where:

$P_{0}=24.700 .000 €$
$\mathrm{T}=5$
$m=1$
$F V=26.000 .000 €:$
$\mathrm{i}=4 \%$
$\mathrm{C}=\mathrm{FV} \times \frac{\mathrm{i}}{\mathrm{m}}=1.040 .000 €:$
the market value of the bond
the bond's maturity in years
coupons per year
the face value of the bond
the coupon rate
the coupon

The cost of the preferred shares is as follows:

| $e^{e^{e^{e^{x^{2}}}}}$ | issue price | 40,00 € |
| :---: | :---: | :---: |
|  | face value | 18.000.000,00 € |
|  | number of shares | 450.000 |
|  | dividend yield on par | 12,00\% |
|  | dividends per annum | 1 |
|  | flotation costs | 360.000,00 € |
|  | flotation costs (\%) | 2,00\% |
|  | preferred dividend | 2.160.000,00 € |
|  | cost of preferred shares | 12,24\% |
| $\mathrm{k}_{\mathrm{PS}}=$ | $\frac{\mathrm{P}_{\text {nominal }} \times \mathrm{d}}{\text { et issue price today }}=\frac{4( }{40 €}$ | $\frac{}{\%)}=12,24 \%$ |

The cost of common equity, calculated through DDM and CAPM is as follows:


| $0_{0} x_{0}^{x_{2}}$ | market price | 50,00 € |
| :---: | :---: | :---: |
|  | current dividend | 2,50 € |
|  | growth rate (g) | 8,00\% |
|  | beta | 1,4 |
|  | risk free rate | 4,00\% |
|  | market risk premium | 8,00\% |
|  | market portfolio expected return | 12,00\% |
|  | cost of common equity (ddm) | 13,40\% |
|  | cost of common equity (capm) | 15,20\% |
|  | cost of common equity (average | 14,30\% |

$$
k_{C S}=\frac{D_{0} \times(1+g)}{P_{0}}+g=\frac{2,50 € \times(1+8 \%)}{50 €}+8 \%=13,40 \%
$$

and

$$
\mathrm{k}_{\mathrm{CS}}=\mathrm{r}_{\mathrm{f}}+\left[\mathrm{E}\left(\mathrm{r}_{\mathrm{m}}\right)-\mathrm{r}_{\mathrm{f}}\right] \times \text { beta }=4 \%+8 \% \times 1,40=15,20 \%
$$

The weights (in book value terms) of each of the funding schemes in the overall financing is as follows:

| $x^{x^{e^{2}}}$ |  | Market values | Weights |
| :---: | :---: | :---: | :---: |
|  | zero coupon bond | 25.600.000,00 € | 25,60\% |
|  | bond | 24.700.000,00 € | 24,70\% |
|  | preferred stock | 17.640.000,00 € | 17,64\% |
|  | common stock (CAPM) | 32.060.000,00 € | 32,06\% |
|  | Totals | 100.000.000,00 € | 100,00\% |

As such the WACC under the DDM model is:

$$
\begin{aligned}
\text { WACC }_{\text {DDM }}= & 25,60 \% \times 2,97 \%+24,70 \% \times 3,35 \%+17,64 \% \times 12,24 \% \\
& +32,06 \% \times 13,40 \%=7,28 \%
\end{aligned}
$$

And the WACC under the CAPM is:

$$
\begin{aligned}
\text { WACC }_{\text {CAPM }}= & 25,60 \% \times 2,97 \%+24,70 \% \times 3,35 \%+17,64 \% \times 12,24 \% \\
& +32,06 \% \times 15,20 \%=7,86 \%
\end{aligned}
$$



The WACC differs between the two methods, although not severely, since the two methods (DDM and CAPM) lead to different estimates of the cost of equity. We believe that it is inevitable to have such divergences.

DDM will estimate the cost of equity based on current share price and on assumptions about future dividend growth.

CAPM will base its estimate on current beta coefficient and expectations about market return.

So different estimates are not a surprise. However, if these estimates pose a significant divergence, we should check our assumptions.

A conservative management should pick the method leading to a higher WACC (less favorable). A higher WACC practically means that a higher hurdle rate is used for investment projects appraisal. In our case this would be the WACC that is calculated when using the CAPM model to estimate the cost of common equity (i.e. $W A C C=7,86 \%)$.

## Part B

Theoretically we could decrease WACC by boosting debt financing as much as possible, taking advantage of the fact that it is the cheaper source of financing.

However if we increase debt financing too much we run bankruptcy risks, which will eventually be priced in the cost of capital (not only in the cost of debt but also in the cost of equity). This could drive WACC to levels higher than the cost of common equity.

There is an optimal capital structure, which is determined by how the cost of debt reacts to different levels of leverage.

The capital structure theories commenced with the work of Modigliani - Miller in 1958 (MM 1958) ${ }^{2}$. In this paper the authors asserted that capital structure would not affect the value of the company, i.e. the value of a company would be the same whether it is financed solely by equity or by a mixture of equity and debt. This is because the cost of equity will rise as the company bears more debt (becomes more risky for shareholders). However, MM 1958 assumes $^{3}$ that there are no bankruptcy costs and companies can lend or borrow at risk free rate however high the leverage. The relationship of higher cost of equity at higher leverage and the stable cost of debt is such that the weighted average cost of capital remains fixed.

The two propositions of MM 1958 are the following:

$$
\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}
$$

[^1]- Investors and firms can borrow and lend at the same rate
- There are no transaction costs or other obstacles in the free flow of information
- There are no agency costs or costs for financial distress
vi. The lending of forms and investors is riskless and as such the cost of lending is the risk free rate

where:
$\mathrm{V}_{\mathrm{L}} \quad$ the value of the firm that is leveraged
$V_{U} \quad$ the value of the unlevered firm

And,

$$
\mathrm{k}_{\mathrm{sL}}=\mathrm{k}_{\mathrm{sU}}+\left(\mathrm{k}_{\mathrm{sU}}-\mathrm{k}_{\mathrm{d}}\right) \times \frac{\mathrm{D}}{\mathrm{~S}}
$$

where:
$\mathrm{k}_{\mathrm{sL}} \quad$ the cost of equity of the leveraged firm
$\mathrm{k}_{\mathrm{SU}} \quad$ the cost of equity of the unlevered firm
$\mathrm{k}_{\mathrm{d}} \quad$ the cost of debt (equal to the risk free rate)

D the value of debt (third party capital)

S the value of own equity

In a second attempt Modigliani - Miller in 1963 (MM 1963) ${ }^{4}$ relaxed their assumption of no corporate taxes. This leads to the proposals that the value of the levered firm will be higher than that of the unlevered one by the tax advantage of the interest expense, and that the WACC will be lower as the firm assumes more leverage. That is:

$$
V_{L}=V_{U}+T_{C} \times D
$$

'О $\pi 0 v:$
$\mathrm{V}_{\mathrm{L}} \quad$ the value of the firm that is leveraged
$V_{U} \quad$ the value of the unlevered firm

[^2]
$\mathrm{T}_{\mathrm{C}} \quad$ the tax rate of the firm

D the value of debt (third party capital)

And,

$$
\mathrm{k}_{\mathrm{sL}}=\mathrm{k}_{\mathrm{sU}}+\left(\mathrm{k}_{\mathrm{sU}}-\mathrm{k}_{\mathrm{d}}\right) \times \frac{\mathrm{D}}{\mathrm{~S}} \times\left(1-\mathrm{T}_{\mathrm{C}}\right)
$$

where:
$\mathrm{k}_{\mathrm{sL}} \quad$ the cost of equity of the leveraged firm
$k_{s U} \quad$ the cost of equity of the unlevered firm
$\mathrm{k}_{\mathrm{d}} \quad$ the cost of debt (equal to the risk free rate)

D the value of debt (third party capital)

S the value of own equity
$\mathrm{T}_{\mathrm{C}} \quad$ the tax rate of the firm

The third attempt by Miller $1977^{5}$, includes also investors' taxes and results in the following proposition:

The value of the levered firm is:

$$
\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\left[1-\frac{\left(1-\mathrm{T}_{\mathrm{C}}\right) \times\left(1-\mathrm{T}_{\mathrm{S}}\right)}{1-\mathrm{T}_{\mathrm{D}}}\right] \times \mathrm{D}
$$

'O $\pi$ ov:
$\mathrm{V}_{\mathrm{L}} \quad$ the value of the firm that is leveraged
$V_{U} \quad$ the value of the unlevered firm
$\mathrm{T}_{\mathrm{C}} \quad$ the tax rate of the firm

[^3]$\mathrm{T}_{\mathrm{S}} \quad$ the tax rate of investors for income coming from shares
$T_{D} \quad$ the tax rate of investors for income coming from bonds

D the value of debt (third party capital)

In other word, the value of the firm is subject to the relation of the various tax rates.

However, the Modigliani - Miller assumptions are too restrictive, and thus not realistic, especially the absence of financial distress costs. As such, the trade - off model arose, which claims that there is an optimal leverage ratio, which if surpassed significant distress costs will raise the cost of debt along with the cost of equity, leading to a higher WACC . That is the relation of WACC to leverage ratio is a U shape. However, the trade-off theory will not discriminate on sources of equity, treating financing through retained earnings the same as via issuance of new equity.

The pecking order theory ${ }^{6}$ is based on the existence of asymmetric information; the latter points that managers know more about their companies prospects, risks and value than outside investors.

Proponents of the pecking order theory underline that the firm should prioritize its sources of financing, first preferring internal financing, and then debt, lastly raising equity as a last resort. In that essence the existence of asymmetric information ${ }^{7}$ favors the issue of debt over equity as the issue of debt signals the management's confidence that an investment is profitable and that the current stock price is undervalued; an issuance of equity would signal a lack of confidence in the management team and that they feel the share price is over-valued. An issue of equity would therefore lead to a drop in share price.

[^4]
## LITERATURE

 Практькท́, Avavєшน
2. Brealey A. R., Myers C. S., (2000), Principles of Corporate Finance, 6e, McGraw-Hill.
3. Copeland E. T., Weston J. F., (1992), Financial Theory and Corporate Policy, 3e, Addison-Wesley
4. Damodaran A., (1999), Applied Corporate Finance: A User's Manual, John Wiley \& Sons Inc.
5. Ross A. S., Westerfield W. R., Jaffe J., (2013), Corporate Finance, 10e, McGraw-Hill/Irwin.


[^0]:    ${ }^{1}$ By the term "incremental" we mean all the amounts of the elements of the income statement that are additional to the income statement of the company if the latter would not proceed with the project.

[^1]:    ${ }^{2}$ Modigliani, F.; Miller, M. (1958). "The Cost of Capital, Corporation Finance and the Theory of Investment". American Economic Review. 48 (3): 261-297
    ${ }^{3}$ Modigliani - Miller based their theories on a series of assumptions, which do not hold in real life. The assumptions are the following:
    i. All firms have the same business risk.
    ii. Investors have homogeneous expectations concerning the future earnings of the firms and their risk.
    iii. All cash flows are perpetual (zero growth rate and stable earnings)
    iv. There are no taxes involved
    v. Stocks and bonds are listed in perfect capital markets.

[^2]:    ${ }^{4}$ Modigliani, F.; Miller, M. (1963). "Corporate income taxes and the cost of capital: a correction". American Economic Review. 53 (3): 433-443

[^3]:    ${ }^{5}$ Miller, M. H. (1977), DEBT AND TAXES. The Journal of Finance, 32: 261-275

[^4]:    ${ }^{6}$ Donaldson G., (1961), Corporate Debt Capacity: A study of Corporate Debt Policy and the Determination of Corporate Debt Capacity, Harvard Graduate School of Business Administration, Boston
    ${ }^{7}$ Myers S.C., (1984), "The Capital Structure Puzzle", Journal of Finance, July, pp. 575-592.

